Lecture-6

Properties of materials
Resistance and capacitance

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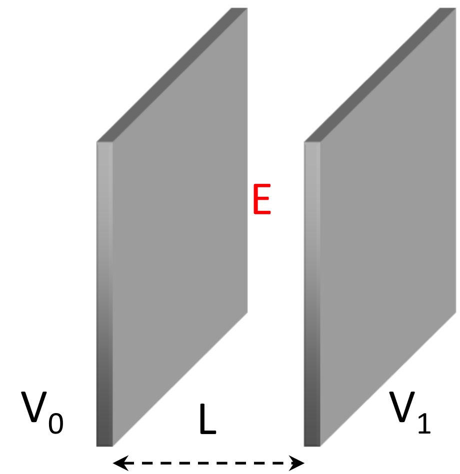
Capacitance and Capacitors

Capacitance

A capacitor is basically two parallel conducting plates with air or insulating material in between.

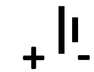


A capacitor doesn't have to look like metal plates.

Capacitor for use in high-performance audio systems.

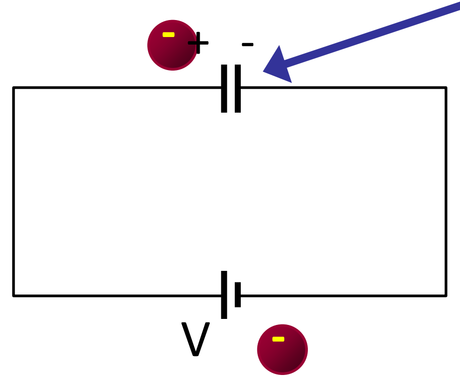
The symbol representing a capacitor in an electric circuit looks like parallel plates.

Here's the symbol for a battery, or an external potential.



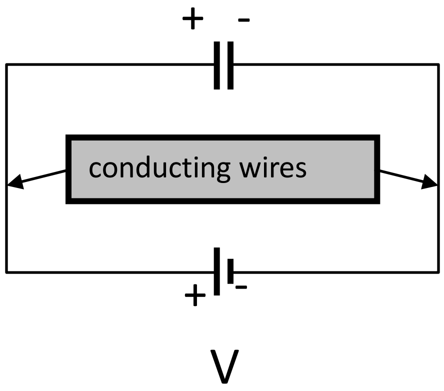
When a capacitor is connected to an external potential, charges flow onto the plates and create a potential difference between the plates.

Capacitor plates build up charge.



The battery in this circuit has some voltage V. We haven't discussed what that means yet.

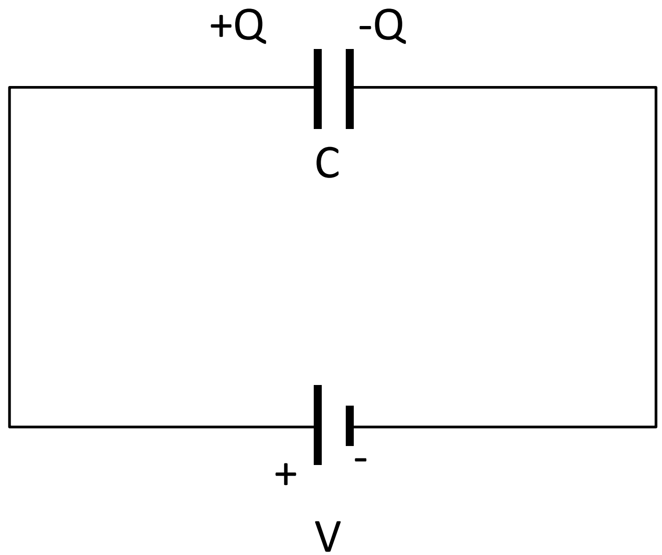
If the external potential is disconnected, charges remain on the plates, so capacitors are good for storing charge (and energy).



Capacitors are also very good at releasing their stored charge all at once. The capacitors in your tube-type TV are so good at storing energy that touching the two terminals at the same time can be fatal, even though the TV may not have been used for months.

High-voltage TV capacitors are supposed to have "bleeder resistors" that drain the charge away after the circuit is turned off. I wouldn't bet my life on it.

assortment of capacitors



Here's this V again. It is the potential difference provided by the "external potential." For example, the voltage of a battery.

The magnitude of charge acquired by each plate of a capacitor is Q=CV where C is the capacitance of the capacitor.

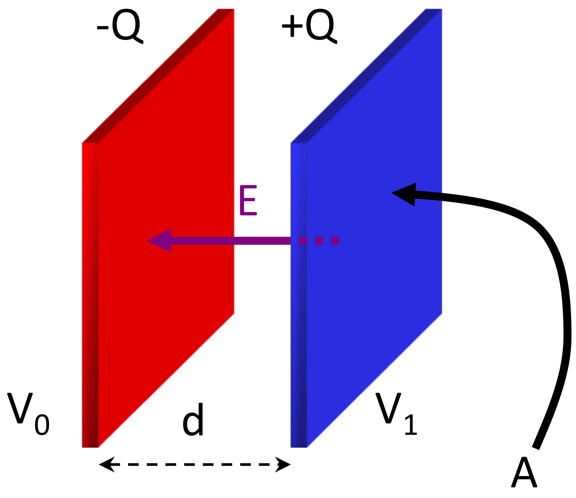
$$C = \frac{Q}{V}$$

C is always positive.

The unit of C is the farad but most capacitors have values of C ranging from picofarads to microfarads (pF to μF).

Parallel Plate Capacitance

We previously calculated the electric field between two parallel charged plates:



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \ .$$

This is valid when the separation is small compared with the plate dimensions.

We also showed that E and ΔV are related:

$$\Delta V = -\int_0^d \vec{E} \cdot d\vec{\ell} = E\int_0^d dx = Ed \ .$$

This lets us calculate C for a parallel plate capacitor.

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{\left(\frac{Q}{\varepsilon_0 A}\right)d} = \frac{\varepsilon_0 A}{d}$$

Reminders: $$C = \frac{Q}{V}$$

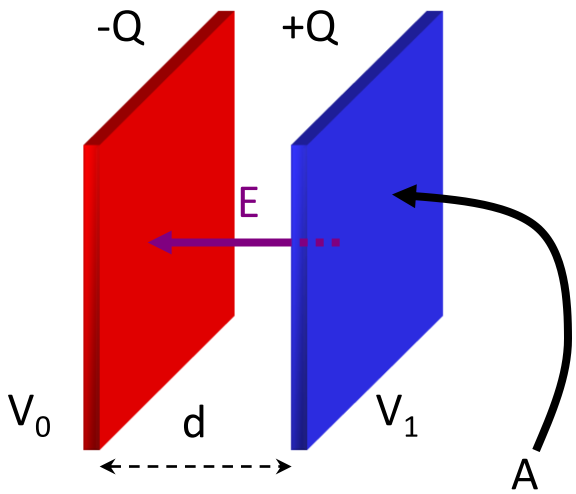
Q is the magnitude of the charge on either plate.

V is actually the magnitude of the potential difference between the plates. V is really $|\Delta V|$. Your book calls it V_{ab}.

C is always positive.

Parallel plate capacitance depends "only" on geometry.

$$C = \frac{\varepsilon_0 A}{d}$$



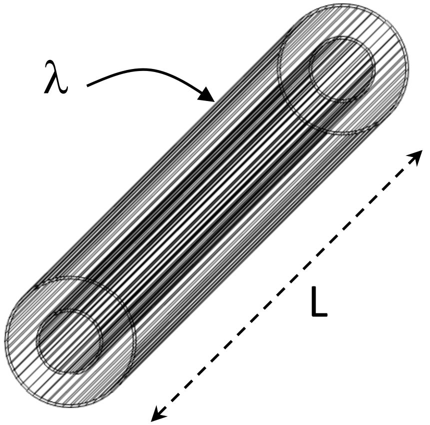
This expression is approximate, and must be modified if the plates are small, or separated by a medium other than a vacuum (lecture 9).

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

Greek letter Kappa. For today's lecture (and for exam 1), use Kappa=1.

Coaxial Cylinder Capacitance

We can also calculate the capacitance of a cylindrical capacitor (made of coaxial cylinders).



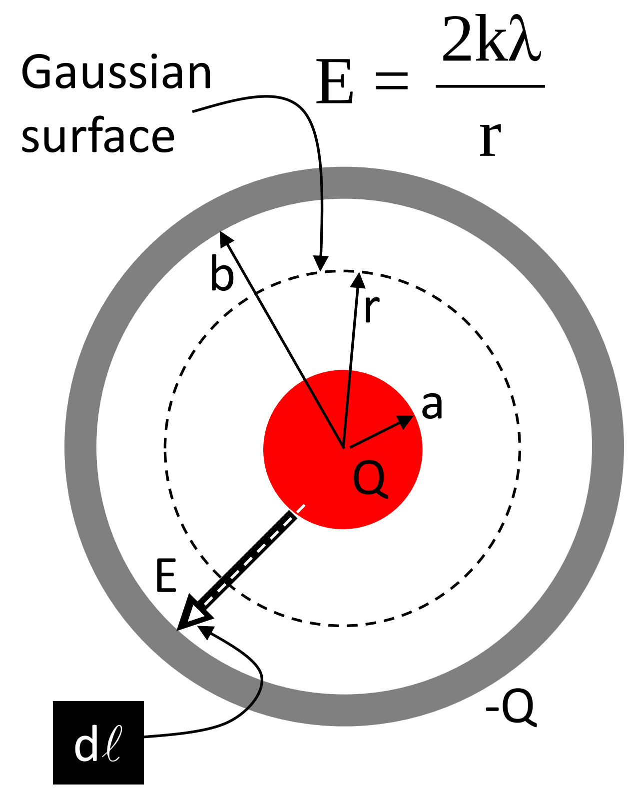
The next slide shows a cross-section view of the cylinders.

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell} = -\int_a^b E_r \, dr$$

$$\Delta V = -2k\lambda \int_a^b \frac{dr}{r} = -2k\lambda \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{|\Delta V|} = \frac{\lambda L}{2k\lambda \ln\left(\frac{b}{a}\right)}$$

$$C = \frac{L}{2k \ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$



Lowercase c is capacitance per unit length:

$$c = \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}$$

Isolated Sphere Capacitance

An isolated sphere can be thought of as concentric spheres with the outer sphere at an infinite distance and zero potential.

We already know the potential outside a conducting sphere:

$$V = \frac{Q}{4\pi\varepsilon_0 r} .$$

The potential at the surface of a charged sphere of radius R is

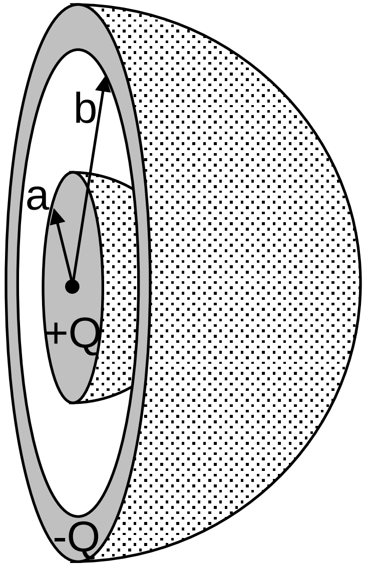
$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

so the capacitance at the surface of an isolated sphere is

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 R.$$

Capacitance of Concentric Spheres

Let's calculate the capacitance of a concentric spherical capacitor of charge Q



In between the spheres

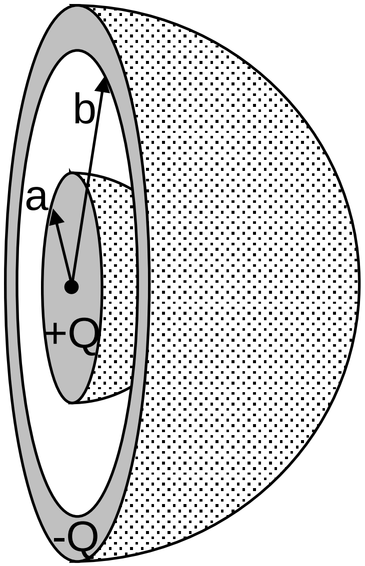
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$|\Delta V| = \frac{Q}{4\pi\varepsilon_0} \int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

alternative calculation of capacitance of isolated sphere

$$C = \frac{Q}{\Delta V} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$



Let a→R and b→∞ to get the capacitance of an isolated sphere.

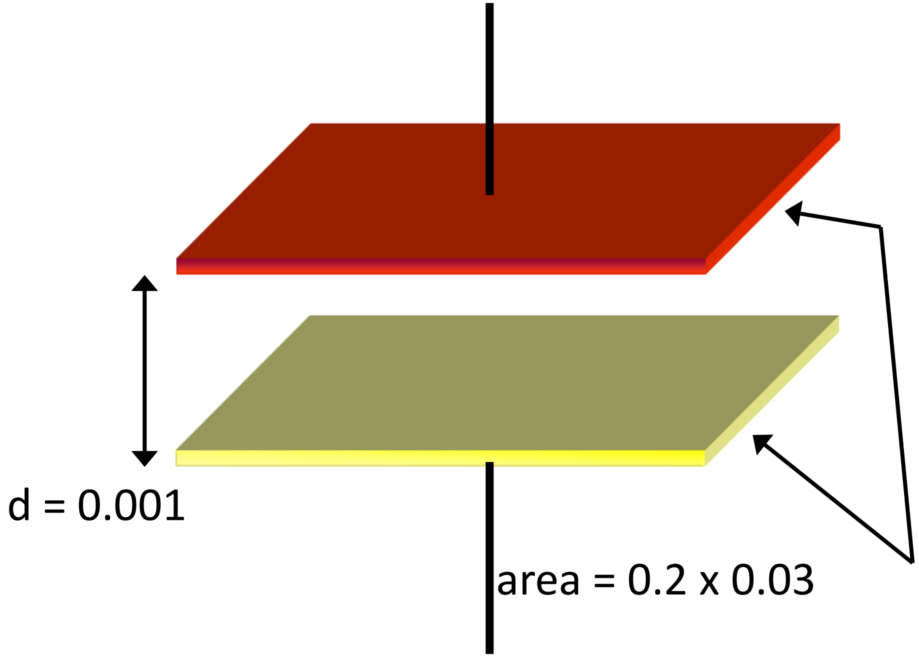
Example: calculate the capacitance of a capacitor whose plates are 20 cm x 3 cm and are separated by a 1.0 mm air gap.

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\left(8.85\times10^{-12}\right)\left(0.2\times0.03\right)}{0.001}$$

$$C = 53\times10^{-12}\ \text{F}$$

$$\boxed{C = 53\ \text{pF}}$$

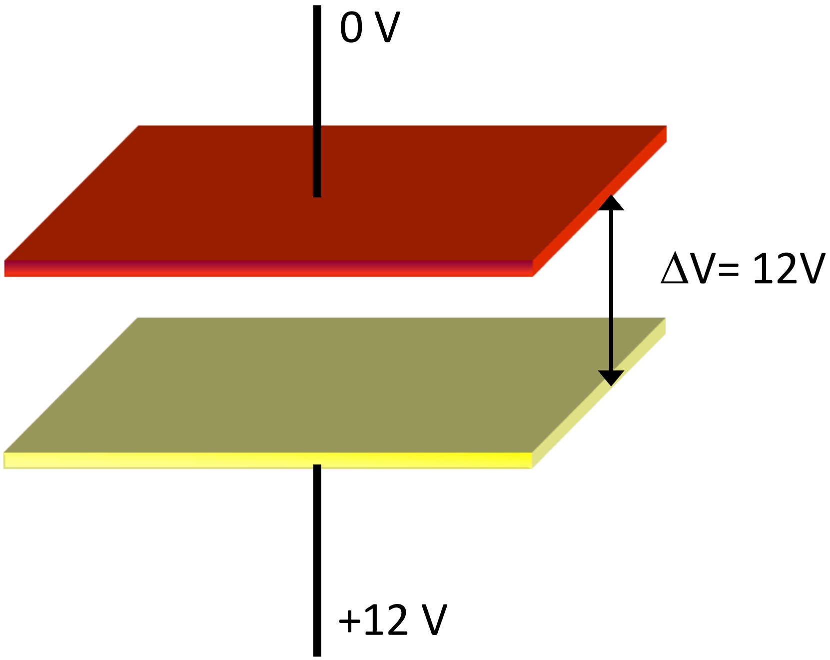


Example: what is the charge on each plate if the capacitor is connected to a 12 volt* battery?

$$Q = CV$$

$$Q = \left(53\times10^{-12}\right)(12)$$

$$\boxed{Q = 6.4\times10^{-10}\ \text{C}}$$

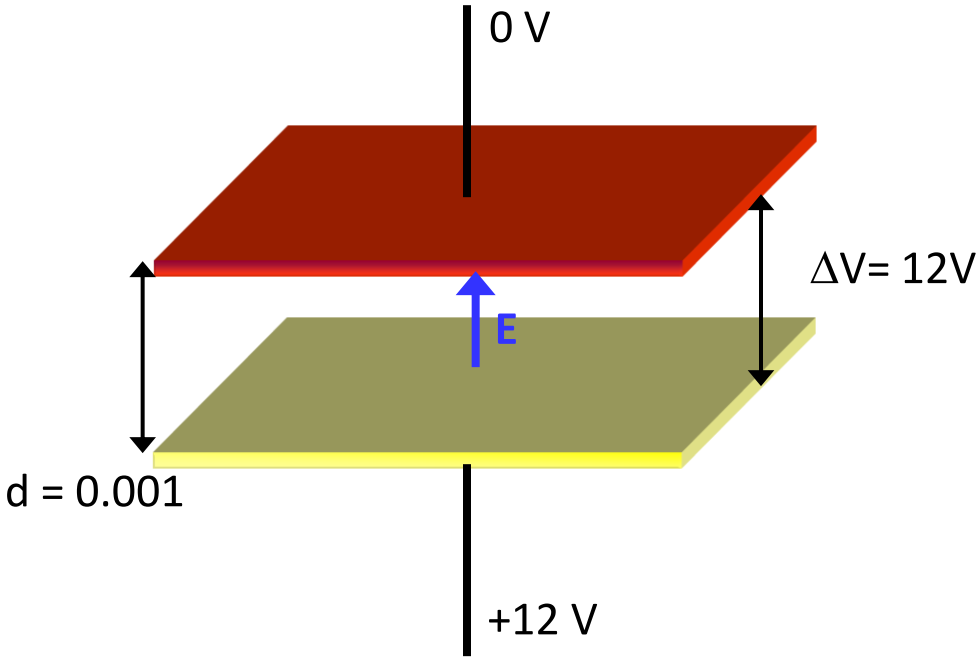


Example: what is the electric field between the plates?

$$E = \frac{\Delta V}{d}$$

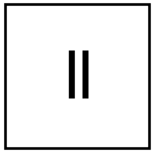
$$E = \frac{12\text{V}}{0.001\text{ m}}$$

$$\boxed{\vec{E} = 12000\ \frac{\text{V}}{\text{m}}\ ,\text{"up."}}$$

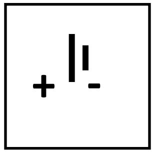


Capacitors in Circuits

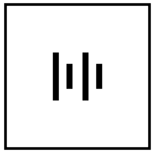
Recall: this is the symbol representing a capacitor in an electric circuit.



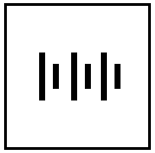
And this is the symbol for a battery…



…or this…

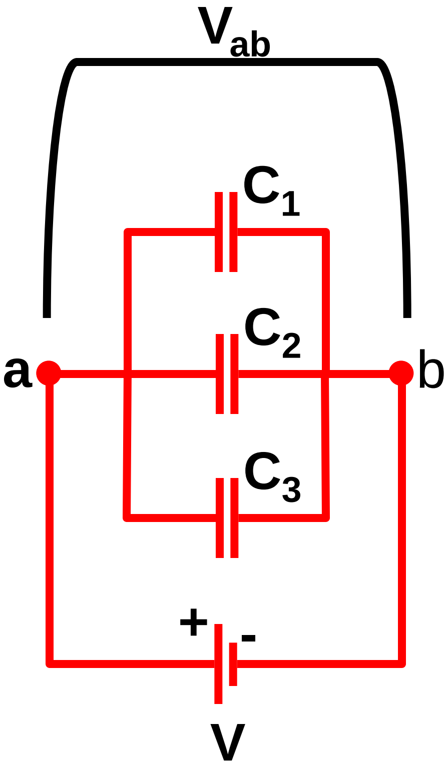


…or this.



Circuits Containing Capacitors in Parallel

Capacitors connected in parallel:



The potential difference (voltage drop) from a to b must equal V.

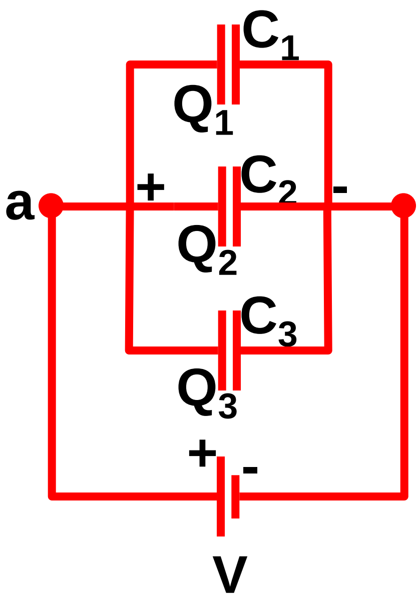
V_{ab} = V = voltage drop across each individual capacitor.

$Q = C\,V$

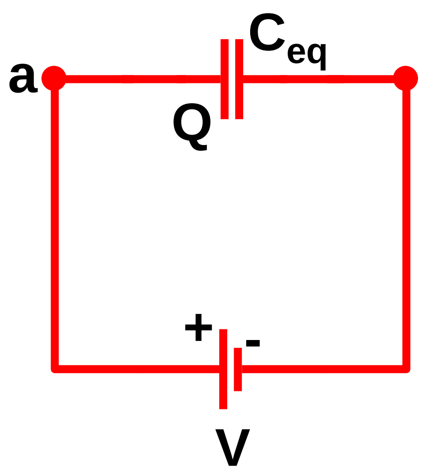
$\Rightarrow Q_1 = C_1\,V$

$\&\quad Q_2 = C_2\,V$

$\&\quad Q_3 = C_3\,V$



Now imagine replacing the parallel combination of capacitors by a single equivalent capacitor.



By "equivalent," we mean "stores the same total charge if the voltage is the same."

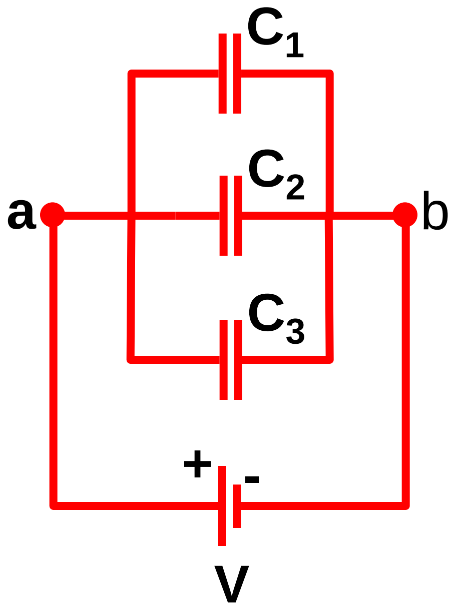
$Q_1 + Q_2 + Q_3 = C_{eq}\,V = Q$

Important!

Summarizing the equations on the last slide:

$Q_1 = C_1\,V \quad Q_2 = C_2\,V \quad Q_3 = C_3\,V$

$Q_1 + Q_2 + Q_3 = C_{eq}\,V$



Using $Q_1 = C_1V$, etc., gives

$C_1V + C_2V + C_3V = C_{eq}\,V$

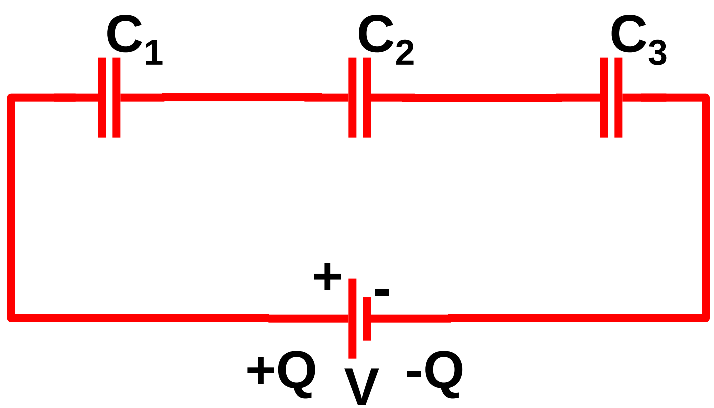
$C_1 + C_2 + C_3 = C_{eq}$ (after dividing both sides by V)

Generalizing:

$C_{eq} = \Sigma C_i$ (capacitors in parallel)

Circuits Containing Capacitors in Series

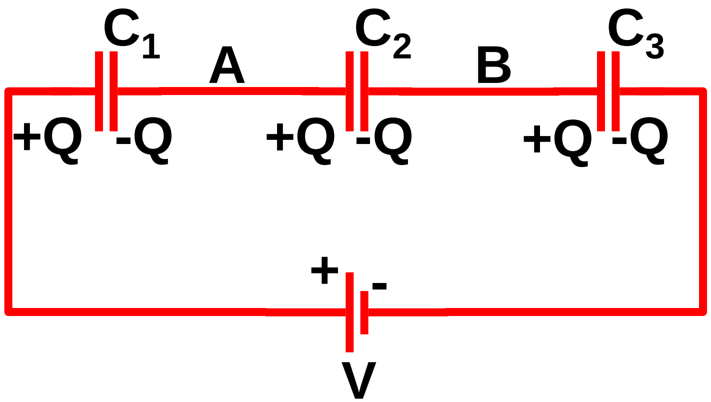
Capacitors connected in series:



An amount of charge +Q flows from the battery to the left plate of C_1. (Of course, the charge doesn't all flow at once).

An amount of charge -Q flows from the battery to the right plate of C_3. Note that +Q and –Q must be the same in magnitude but of opposite sign.

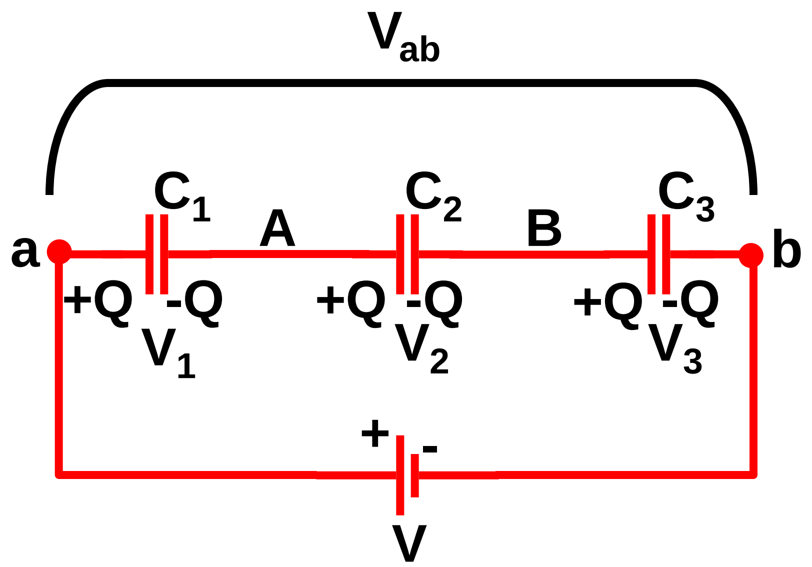
The charges +Q and –Q attract equal and opposite charges to the other plates of their respective capacitors:



These equal and opposite charges came from the originally neutral circuit regions A and B.

Because region A must be neutral, there must be a charge +Q on the left plate of C_2.

Because region B must be neutral, there must be a charge --Q on the right plate of C_2.



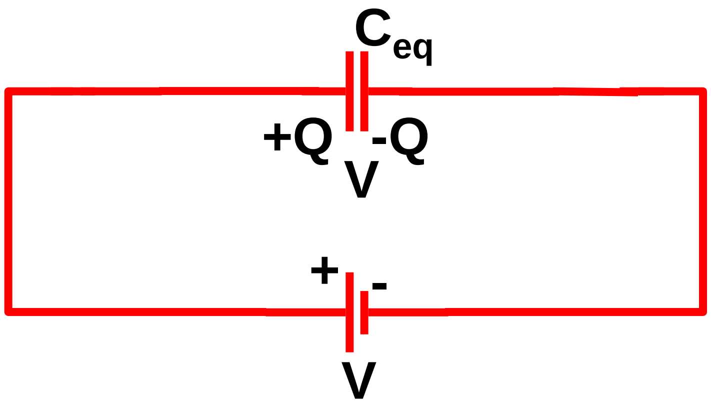
The charges on C_1, C_2, and C_3 are the same, and are

$$Q = C_1 V_1 \quad Q = C_2 V_2 \quad Q = C_3 V_3$$

But we don't know V_1, V_2, and V_3 yet.

We do know that $V_{ab} = V$ and also $V_{ab} = V_1 + V_2 + V_3$.

Let's replace the three capacitors by a single equivalent capacitor.



By "equivalent" we mean V is the same as the total voltage drop across the three capacitors, and the amount of charge Q that flowed out of the battery is the same as when there were three capacitors.

$Q = C_{eq} V$

Collecting equations:

$Q = C_1 V_1$ $Q = C_2 V_2$ $Q = C_3 V_3$

Important!

$V_{\mathbf{ab}} = V = V_1 + V_2 + V_3$.

$Q = C_{eq} V$

Substituting for V_1, V_2, and V_3:

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Substituting for V:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Dividing both sides by Q:

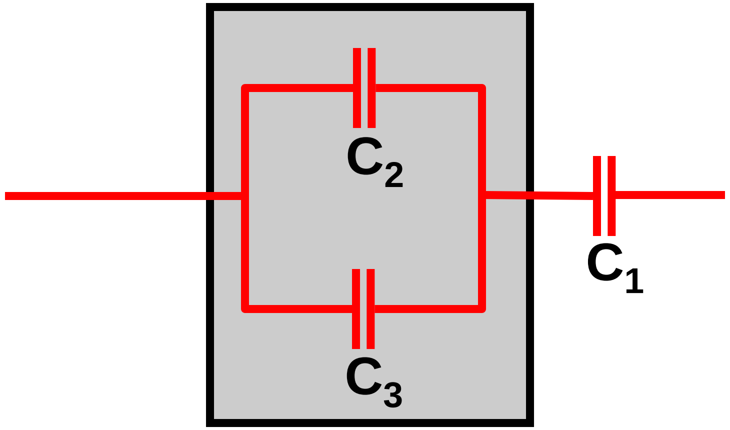
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Generalizing:

OSE:

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$ (capacitors in series)

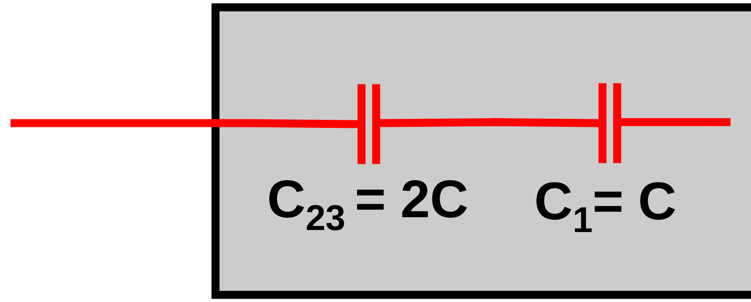
Example: determine the capacitance of a single capacitor that will have the same effect as the combination shown. Use
$C_1 = C_2 = C_3 = C$.



I don't see a series combination of capacitors, but I do see a parallel combination.

$C_{23} = C_2 + C_3 = C + C = 2C$

Now I see a series combination.



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} = \frac{2}{2C} + \frac{1}{2C} = \frac{3}{2C}$$

$$\boxed{C_{eq} = \frac{2}{3}C}$$

Example: for the capacitor circuit shown, C_1 = 3μF, C_2 = 6μF, C_3 = 2μF, and C_4 =4μF. (a) Find the equivalent capacitance. (b) if ΔV=12 V, find the potential difference across C_4.